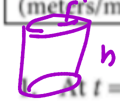
College Board Module

The data in the table below give selected values for the velocity, in meters/minute, of a particle moving along the x-axis. The velocity v is a differentiable function of time t.

Time t (min)	0	2	5	6	8	12
Velocity $v(t)$ (meters/min)	[illegible]	2	3	5	7	5

$V = \pi r^2 \cdot h = 16\pi$

$2\pi r^2 + 2\pi r \cdot h$



$2\pi r^2 + 2\pi \cancel{r} \cdot \frac{16}{r^{\cancel{2}}} = SA$

1. At $t = 0$, is the particle moving to the right or to the left? Explain your answer.

$\frac{\cancel{\pi} r^2 h}{\cancel{\pi}} = \frac{16\cancel{\pi}}{\cancel{\pi}}$

$h = \frac{16}{r^2}$

$2\pi r^2 + 32\pi r^{-1} = SA$

$4\pi r + -32\pi r^{-2} = \frac{dSA}{dx}$

$\frac{\cancel{r^2} h}{\cancel{r^2}} = \frac{16}{r^2}$

$h = \frac{16}{2^2} = \frac{16}{4} = 4$ $\qquad 4\pi r - \frac{32\pi}{r^2} = 0$

2. Is there a time during the time interval $0 \le t \le 12$ minutes when the particle is at rest? Explain your answer.

$r^3 = 8$

$\boxed{r = 2}$

$\frac{r^2 \cdot 4\pi r}{4\pi} = \frac{32\pi}{4\pi} \cdot r^2$

3. Use data from the table to find an approximation for $v'(10)$ and explain the meaning of $v'(10)$ in terms of the motion of the particle. Show the computations that lead to your answer and indicate units of measure.

$y^2 = 2$

$\frac{12\sqrt{2} \cdot \sqrt{2}}{2}$

$4 \cdot \frac{3\sqrt{2}}{2} \cdot \sqrt{2}$

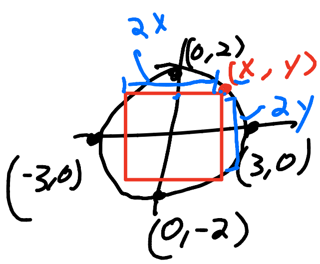
$A = 2x \cdot 2y = 4xy$

$4x^2 + 9 \cdot 2 = 36$

$4x^2 = 18$

$x = \frac{3\sqrt{2}}{2}$

$\frac{4x^2 + 9y^2 = 36}{36 \qquad\quad 36}$



$\frac{4x^2}{36} + \frac{9y^2}{36} = 1$

$\frac{x^2}{9} + \frac{y^2}{4} = 1$

$A = 4\left(\sqrt{\frac{36 - 9y^2}{4}}\right) \cdot y$

$A = \cancel{4}^{2} \cdot \frac{(36 - 9y^2)^{\frac{1}{2}}}{\cancel{2}} \cdot y$

$4x^2 + \cancel{9y^2} = 36 - 9y^2$
$\quad -\cancel{9y^2}$

$A = 2y\,(36 - 9y^2)^{\frac{1}{2}}$

$\frac{dA}{dy} = 2(36 - 9y^2)^{\frac{1}{2}} + \cancel{2}y\left[\frac{-18y}{\cancel{2}(36 - 9y^2)^{\frac{1}{2}}}\right]$

$\frac{\cancel{4}x^2}{\cancel{4}} = \frac{36 - 9y^2}{4}$

$x^2 = \frac{36 - 9y^2}{4} \Rightarrow x = \sqrt{\frac{36 - 9y^2}{4}}$

$y=(36-9x^2)^{\frac{1}{2}} \Rightarrow y=u^{\frac{1}{2}}$

$u=36-9x^2$ $\quad \frac{dy}{du}=\frac{1}{2}u^{\frac{1}{2}-1}=\frac{1}{2}u^{-\frac{1}{2}}=\frac{1}{2u^{\frac{1}{2}}}=\frac{1}{2\sqrt{u}}$

$\frac{du}{dx}=-18x$

$\frac{du}{dx}\cdot\frac{dy}{du}=-18x\cdot\frac{1}{2\sqrt{u}}=\frac{-18x}{2\sqrt{36-9x^2}}$

$0=2\sqrt{36-9y^2}-\frac{18y^2}{\sqrt{36-9y^2}} \Rightarrow \cancel{\sqrt{36-9y^2}}\cdot\frac{18y^2}{\cancel{\sqrt{36-9y^2}}}=2\sqrt{36-9y^2}\cdot\sqrt{36-9y^2}$

$18y^2=2(36-9y^2)$

$18y^2=72-18y^2 \Rightarrow 36y^2=72$

$y^2=2$

9. **2003 #26 (AB but suitable for BC) - No Calc:** What is the slope of the line tangent to the curve $3y^2-2x^2=6-2xy$ at the point $(3,2)$?

Find $\frac{dy}{dx}$

a. 0 c. $\frac{7}{9}$ e. $\frac{5}{3}$

b. $\frac{4}{9}$ d. $\frac{6}{7}$

$6y\frac{dy}{dx}-4x=0-2\cdot y+-2x\cdot\frac{dy}{dx}$

$6(2)\frac{dy}{dx}-4(3)=-2\cdot2+-2\cdot3\frac{dy}{dx} \Rightarrow 12\frac{dy}{dx}-12=-4-6\frac{dy}{dx}$

$18\frac{dy}{dx}=8$

$\frac{dy}{dx}=\frac{8}{18}=\frac{4}{9}$

14. Consider all right circular cylinders for which the sum of the height and the circumference is 30 centimeters. What is the radius of the one with maximum volume?

$\frac{dV}{dr}=0$

A) 3 cm B) 10 cm C) 20 cm D) $\frac{30}{\pi^2}$ cm E) $\frac{10}{\pi}$ cm

$h + 2\pi r = 30$

$h = 30 - 2\pi r$

$V = \pi r^2 h$

$V = \pi r^2(30 - 2\pi r)$

$V = 30\pi r^2 - 2\pi^2 r^3$

$\frac{dV}{dT} = 60\pi r - 2\pi^2 \cdot 3r^2$

$0 = 60\pi r - 6\pi^2 r^2$

$\frac{6\pi^2 r^2}{6\pi^2 r} = \frac{60\pi r}{6\pi^2 r}$

$r = \frac{10}{\pi}$

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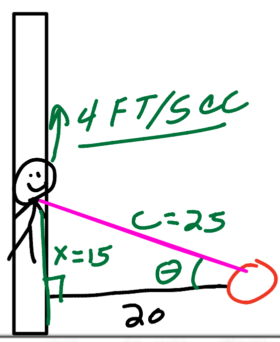
5. A man walks along a straight path at a speed of 4 ft/s. A searchlight is located on the ground 20 ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft from the point on the path closest to the searchlight?

$\cos\theta = \frac{4}{5}$

$\sec\theta = \frac{5}{4}$



$\tan\theta = \frac{x}{20}$

$\frac{d\theta}{dT} = 0$

$\sec^2\theta \cdot \frac{d\theta}{dT} = \frac{1}{20} \cdot \frac{dx}{dx} \Rightarrow \sec^2\theta \frac{d\theta}{dT} = \frac{1}{20} \cdot 4$

$\frac{d\theta}{dT} = \frac{16}{125}$

$\frac{16}{25} \cdot \left(\frac{5}{4}\right)^2 \frac{d\theta}{dT} = \frac{1}{5} \cdot \frac{16}{25}$

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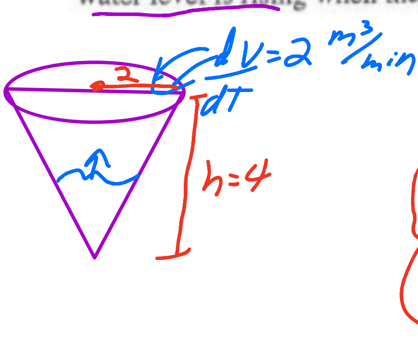
3. A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m. If water is being pumped into the tank at a rate of 2 m3/min, find the rate at which the water level is rising when the water is 3 m deep.



$\frac{r=2}{h=4}$

$r = \frac{h}{2}$

$2r = h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 \cdot h$

$V = \frac{\pi}{12} h^3$

$\frac{dV}{dT} = \frac{\pi}{12} \cdot 3h^2 \frac{dh}{dT} = \frac{\pi}{12} \cdot 3 \cdot 3^2 \frac{dh}{dT}$

$$\frac{dV}{dT} = \frac{9\pi}{4}\frac{dh}{dT}$$

$$\frac{4}{9\pi \cancel{m^2}} \cdot 2\frac{m^3}{min} = \frac{\cancel{9\pi m^2}}{\cancel{4}}\frac{dh}{dT}\cdot\frac{\cancel{4}}{\cancel{9\pi m^2}}$$

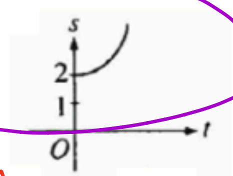
$$\frac{8}{9\pi}\frac{m}{min} = \frac{dh}{dT}$$

$S(0)=2$

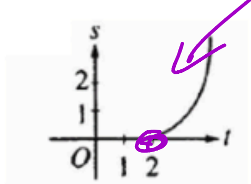
2 away From (0,0)

1. **1998 #90 (BC) - Calc OK:** A particle starts from rest at the point (2, 0) and moves along the x-axis with a constant positive acceleration for time $t \geq 0$. Which of the following could be the graph of the distance $s(t)$ of the particle from the origin as a function of time t?

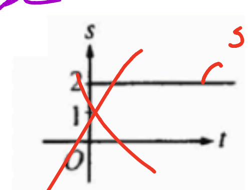
a.



c.



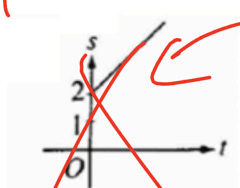
Nope



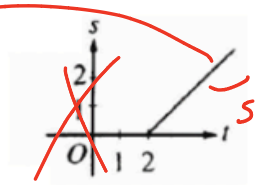
$s'(T)=0$

$S''(T)=0$

b.

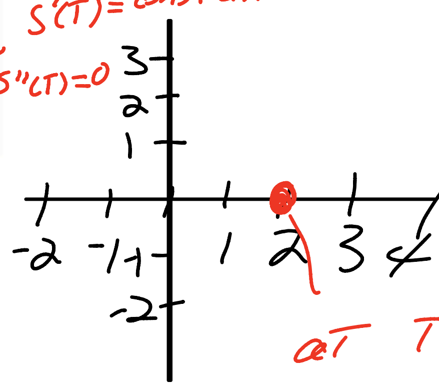


d.



$S'(T)=$ constant

$S''(T)=0$



at Time=0

$S(0)=2$

$(0,2)$

$S''(T)=A(T)=+$

$S(T)$ is concave up

8. **2003 #91 (BC) - Calc OK:** The height h, in meters, of an object at time t is given by $h(t) = 24t + 24t^{\frac{3}{2}} - 16t^2$. What is the height of the object at the instant when it reaches its maximum upward velocity?

a. 2.545 meters
b. 10.263 meters
c. 34.125 meters
d. 54.889 meters
e. 89.005 meters

$h'(t) = V(T) = 24 + 36T^{\frac{1}{2}} - 32T$

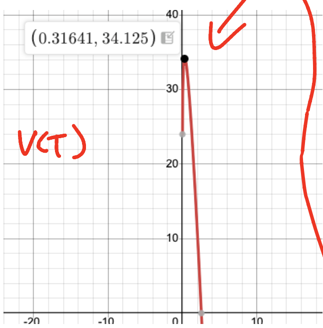
$V'(T) = 0 + 18T^{-\frac{1}{2}} - 32$

$0 = \frac{18}{\sqrt{T}} - 32$

$\cancel{32}\sqrt{T} = \frac{18}{32}$

$\sqrt{T} = \frac{18}{32} = \frac{9}{16}$

$T = \frac{9^2}{16^2} = .316$



V(T)

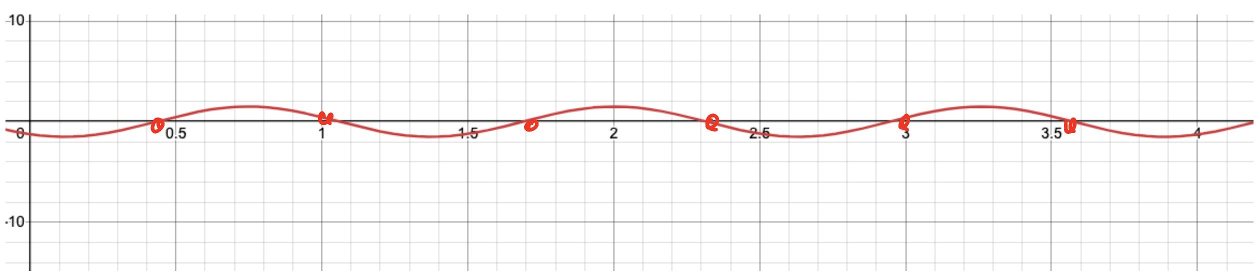
$h(.316) = 24(.316) + 24(.316)^{3/2} - 16(.316)^2$

5. **1997 #79 (BC) - Calc OK:** The position of an object attached to a spring is given by $y(t) = \frac{1}{6}\cos(5t) - \frac{1}{4}\sin(5t)$, where t is time in seconds. In the first 4 seconds, how many times is the velocity of the object equal to 0?

a. Zero
b. Three
c. Five
d. Six
e. Seven

$y'(T) = v(T) = \frac{1}{6} \cdot -\sin(5T) \cdot 5 - \frac{1}{4}\cos(5T) \cdot 5$

$0 = -\frac{5}{6}\sin 5T - \frac{5}{4}\cos 5T$



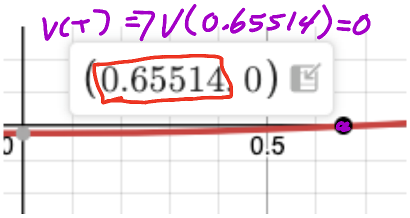
$\int v(T)\,dT = S(T)$

11. **2003 #87 (BC) - Calc OK:** A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = \cos\left(2 - t^2\right)$. The position of the particle is 3 at time $t = 0$. What is the position of the particle when its velocity is first equal to 0?

a. 0.411 b. 1.310 c. 2.816 d. 3,091 e. 3.411

$0 = \cos\left(2 - T^2\right)$

$v(T) \Rightarrow V(0.65514) = 0$



$3 +$

$\int_0^{0.65514} v(T)\,dT = 3 - 0.18354$

Change in Position From $T=0$ To $T=0.6551$

$$y = \int_0^{.65514} \cos\left(2 - t^2\right) dt$$

$= -0.183540627606$

12. **2003 #91 (AB but suitable for BC) - Calc OK:** A particle moves along the x-axis so that at any time $t > 0$, its acceleration is given by $a(t) = \ln\left(1 + 2^t\right)$. If the velocity of the particle is 2 at time $t = 1$, then the velocity of the particle at time $t = 2$ is

a. 0.462 b. 1.690 c. 2.555 d. 2.886 e. 3.346

$\int_1^2 \ln(1+2^T)\,dT$ = Change in Velocity From Time $T=1$ To $T=2$

$$y = \int_1^2 \ln\left(1 + 2^t\right) dt$$

$= 1.34631353382$

$2 + \int_1^2 \ln(1+2^T)\,dT = 3.346$

13. **1997 #87 (AB but suitable for BC) - Calc OK:** At time $t \geq 0$, the acceleration of a particle moving on the x-axis is $a(t) = t + \sin t$. At $t = 0$, the velocity of the particle is -2. For what value of t will the velocity of the particle be zero?

a. 1.02 b. 1.48 c. 1.85 d. 2.81 e. 3.14

$\int a(T)dT = V(T)$

$\int (T + \sin T)dT = \frac{1}{2}T^2 - \cos T + c = V(T)$

$V(0) = -2 = \frac{1}{2}T^2 - \cos T + c$

$\frac{1}{2}T^2 - \cos T + c = V(T)$

$V(0) = -2 = \frac{1}{2}(0)^2 - \cos 0 + c$

$-2 = 0 - 1 + c$

$c = -1$

$\int (T + \sin T)dT = V(T)$

$V(T) = \frac{1}{2}T^2 - \cos T - 1$

10. **1998 #24 (AB but suitable for BC) - No Calc:** The maximum acceleration attained on the interval $0 \leq t \leq 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

a. 9 b. 12 c. 14 d. 21 e. 40

$V'(T) = a(T) = 3T^2 - 6T + 12$

Max/min of $a(T)$

$a'(T) = 0 = 6T - 6$

$T = 1$

T	$a(T) = 3T^2 - 6T + 12$
0	$12 = 3(0)^2 - 6(0) + 12$
1	$9 = 3(1)^2 - 6(1) + 12$
3	$21 = 3(3)^2 - 6(3) + 12$
	$27 - 18 + 12$

2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin(0.4t^2)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$. $-2.118 = \frac{d}{dx}[v(x)]$ at $x=3$

(b) Find the position of the particle at time $t = 3$. $-5 + \int_0^3 v(T)\,dT$

(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

Position

distance

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

(b)

$$\int_0^3 \frac{10\sin(0.4x^2)}{x^2 - x + 3}\,dx$$

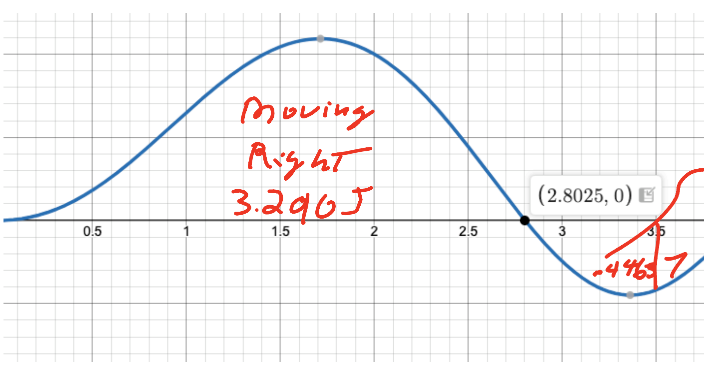
$= 3.2397868128$

$-5 + 3.2397 =$

(c)

Position $3.2905 - .446568$

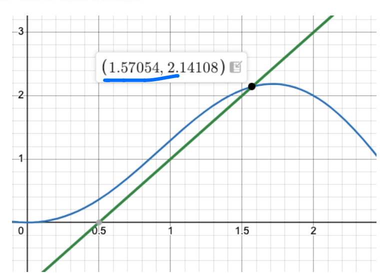
distance $3.2905 + .446568$



Moving Left

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

$V = 2T - 1$



College Board Module

The data in the table below give selected values for the velocity, in meters/minute, of a particle moving along the x-axis. The velocity v is a differentiable function of time t.

Time t (min)	0	2	5	6	8	12
Velocity $v(t)$ (meters/min)	-3	2	3	5	7	5

1. At $t = 0$, is the particle moving to the right or to the left? Explain your answer.

$V(0) = -3$ LefT

2. Is there a time during the time interval $0 \le t \le 12$ minutes when the particle is at rest? Explain your answer.

IVT

Slope of $v(T)$ at $T = 10$

3. Use data from the table to find an approximation for $v'(10)$ and explain the meaning of $v'(10)$ in terms of the motion of the particle. Show the computations that lead to your answer and indicate units of measure.

$(8, 7)\ (12, 5)$

$a(10) = V'(10) \approx -\frac{1}{2}$

$\frac{7-5}{8-12} = \frac{2}{-4} = -\frac{1}{2}$

2017

5. Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$.

(a) For $0 \le t \le 8$, when is particle P moving to the left? $V_P(T) = X_P'(T) = \frac{1}{T^2 - 2T + 10} \cdot 2T - 2$

(b) For $0 \le t \le 8$, find all times t during which the two particles travel in the same direction.

(c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

(d) Find the position of particle Q the first time it changes direction.

(a) $V_P(T) = \frac{2T-2}{T^2 - 2T + 10}$ when is $2T - 2 = -$

Always +

Vertex of Parabola occures $\frac{-b}{2a} = \frac{2}{2} = 1$

$T < 1$ Moves To The LefT

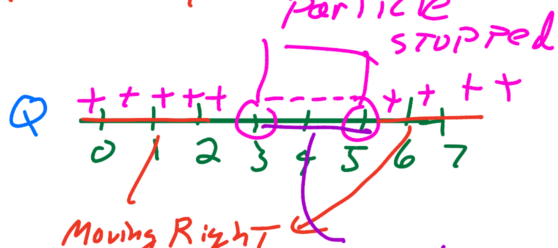
(b) ParTicle P move LefT $0 \le T < 1$

RighT $T > 1$

$V_Q(T) = T^2 - 8T + 15 = (T-5)(T-3)$

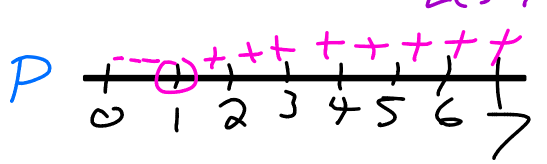
Test PoinTs

T	
0	$+ = (-5)(-3)$
4	$- = (-1)(1)$
6	$+ = 1 \cdot 3$



Same direcTion

$1 < T < 3$ and $5 < T < 8$



(c) $v_Q(t) = t^2 - 8t + 15$.

$a_Q(T) = 2T - 8$

$a(2) = 2 \cdot 2 - 8 = -4$

$V(2) = 2^2 - 8(2) + 15 = 3$

Slowing down
$a(2)$ and $V(2)$
Have opposite

Sig 4S

d) $5+\int_0^3 (T^2-8T+15)\,dT = \underbrace{\frac{1}{3}T^3-4T^2+15T}\, + C \Big]_0^3 + 5$

$\frac{1}{3}(3)^3 - 4(3)^2 + 15(3) - \left[\cancel{\frac{1}{3}(0)^3 - 4(0)^2 + 15(0)}\right]$

$9-36+45 = 18$

$5+18 = 23$